

Revision Exercise for Home Test 1

Question 1. Suppose that $\lim a_n = 3$. Using the definition of limit, show that

$$\lim_{n \rightarrow \infty} \frac{a_n^2 + 1}{a_n - 2} = 10.$$

Question 2. Let (x_n) be a sequence of non-negative real numbers. Suppose that

$$\lim_{n \rightarrow \infty} (-1)^n x_n$$

exists in \mathbb{R} . Show that (x_n) converges and find its limit.

Question 3. Let (x_n) be a bounded sequence of real numbers. Define

$$E = \{x \in \mathbb{R} : \text{there is a subsequence } (x_{n_k}) \text{ of } (x_n) \text{ that converges to } x.\}$$

Let $\alpha = \overline{\lim} x_n$. Show that $\alpha \in E$ and $\alpha = \sup E$.

Question 4. Let a be a positive real number and $x_1 > \sqrt{a}$. Define the sequence (x_n) by

$$x_{n+1} = \frac{1}{2} \left(x_n + \frac{a}{x_n} \right).$$

Prove that (x_n) is convergent and $\lim x_n = \sqrt{a}$.

Question 5. .

- (a) State the **Bolzano-Weierstrass Theorem**.
- (b) State the the **Nested Intervals Property**.
- (c) Using the **Bolzano-Weierstrass Theorem**, prove the **Nested Intervals Property**.

Remark. In the textbook, Bolzano-Weierstrass Theorem is proved using the Nested Intervals Property.